KALMAN FILTER BASED LOCALIZATION AND TRACKING ESTIMATION FOR FINE-SCALE RFID SYSTEMS

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Outline

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• Kalman Filter Based Estimation
  ✓ Variable-gain Kalman filter 1
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MOTIVATION: FINE-SCALE RFID MOTION CAPTURE & LOCALIZATION SYSTEMS
Remote motion capture, and tracking in robotic wayfaring, virtual reality.

Body movement detection by combining passive UHF RFID with Support Vector Machine classification technique together [Amendola 2015].

Among all RFID localization and tracking systems, hybrid schemes have the highest accuracy (5-400 cm) and largest range (hundreds of meters) by combining a variety of sources.

Fig. 1 Movement Detection by Synergizing Passive RFID and Machine Learning [Amendola 2015]
HYBRID INERTIAL MICROWAVE REFLECTOMETRY (HIMR) RFID SYSTEM SET-UP

Fig. 2 Set up of HIMR RFID Localization and Tracking (L&T) System [Akbar 2015]
KALMAN FILTER BASED ESTIMATION FOR HIMR RFID SYSTEM
Fig. 3. An overview of how data is processed. RF sensor and physical measurements are converted to noisy estimates of tag position as well as first and second derivatives of position. Then the Kalman filter produces a refined estimate of position as a function of time.
A tagged object moves in one dimension. Its state-space model is:
\[
\dot{x}(t) = Ax(t) + Bw(t) \tag{1}
\]
\[
y(t) = Cx(t) + v(t) \tag{2}
\]
Application of Newton’s law leads to the coefficient matrices:
\[
A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}
\]
Expected value of the initial state and initial covariance are:
\[
E\{x(0)\} = \hat{x}_0 \tag{4}
\]
\[
E\{(x(0) - \hat{x}_0)(x(0) - \hat{x}_0)\}' = P_0 \tag{5}
\]
The disturbance input and noise input are uncorrelated zero-mean white Gaussian noise with spectral densities \(Q\) and \(R\):
\[
E\{w(t)w'(\tau)\} = Q\delta(t - \tau) \tag{6}
\]
\[
E\{v(t)v'(\tau)\} = R\delta(t - \tau) \tag{7}
\]
TAG MOTION DYNAMICS-DISCRETE TIME

• Estimation is preferably implemented in discrete-time in a digital processor.
• Suppose the values of the continuous signals at the sampling instants are $x_k = x(kT)$, $w_k = w(kT)$, $y_k = y(kT)$, $v_k = v(kT)$ where $T$ is the sampling period. The discrete time state-space model is:

$$x_{k+1} = Fx_k + Gw_k$$  \hspace{1cm} (8)

$$y_k = Hx_k + v_k$$  \hspace{1cm} (9)

where $F = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}$, $G = \begin{bmatrix} 0 \\ 0 \\ T \end{bmatrix}$, $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  \hspace{1cm} (10)

The motion is induced by the finite rectangular-pulse disturbance input $w$ as shown in Figure 4:

Fig. 4 Forcing function $w$ used to introduce motion each rectangular pulse has a duration of 1ms
Inferred from the continuous model, the expected value of the initial state and initial covariance are:

\[
\begin{align*}
E\{x_0\} &= \hat{x}_0 & (11) \\
E\{(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)\}' &= P_0 & (12)
\end{align*}
\]

Where \( \hat{x}_0 \) and \( P_0 \) come from (4)-(5). The disturbance input and noise input satisfy:

\[
\begin{align*}
E\{w(t)w'(\tau)\} &= W & (13) \\
E\{v(t)v'(\tau)\} &= V & (14)
\end{align*}
\]

Assuming small \( T \)

\( W \approx Q/T, V \approx R \) (15)
VARIABLE-GAIN KALMAN FILTER 1
FOR HIMR RFID SYSTEM
VARIABLE-GAIN KALMAN FILTER 1

Using discrete model (8)-(9), the estimate $x_k$ depend on measurement up through the present value $y_k$. The time-varying gain is:

$$L_k = P_k^- H'(HP_k^- H' + V)^{-1}$$  (16)

A posteriori estate estimate and error covariance are:

$$\hat{x}_k^+ = \hat{x}_k^- - L_k (H\hat{x}_k^- - y_k)$$  (17)
$$P_k^+ = (I - L_k H)P_k^-$$  (18)

A prior state estimate and error covariance are:

$$\hat{x}_{k+1}^- = F\hat{x}_k^+$$  (19)
$$P_{k+1}^- = FP_k^+ F' + GWG'$$  (20)
VARIABLE-GAIN KALMAN FILTER 1

Initialization from (11)-(12)
\[
\hat{x}_0^- = \hat{x}_0 \\
P_0^- = P_0
\]

(21) 
(22)

The mean square error
\[
\epsilon_k = E\{(x_k - \hat{x}_k^+)')(x_k - \hat{x}_k^+ )\}
\]

(23)

At time k, the error is determined by \( \hat{x}_0^-, P_0^- \) and the gain sequence \( L_0, ..., L_k \), influenced by \( W \) and \( V \).

By choosing \( L_k \) according to (16), (18), and (20), \( \epsilon_k \) is minimized for every \( k \geq 0 \).
VARIABLE-GAIN KALMAN FILTER 2
FOR HIMR RFID SYSTEM
Using discrete model (8)-(9), the estimate $x_k$ depend on measurement up through the present value $y_{k-1}$. The time-varying gain is:

$$L_k = FP_k H' (HP_k H' + V)^{-1}$$

(24)

A estate estimate and error covariance are:

$$\hat{x}_{k+1} = F \hat{x}_k - L_k (H \hat{x}_k - y_k)$$

(25)

where $P_{k+1} = (F - L_k H) P_k^+ F' + GWG'$

(26)

Initialization from (11)-(12)

$$\hat{x}_0 = \hat{x}_0 \quad P_0 = P_0$$

(27)

The mean square error $\epsilon_k = E\{ (x_k - \hat{x}_k^+)'(x_k - \hat{x}_k^+)\}$

(28)

At time $k$, the error is determined by $\hat{x}_0^-, P_0^-$ and the gain sequence $L_0, ..., L_k$, influenced by W and V.

By choosing $L_k$ according to (24) and (26), $\epsilon_k$ is minimized for every $k \geq 0$. 
FIXED-GAIN KALMAN FILTER
FOR HIMR RFID SYSTEM
Using continuous model (1)-(2), the estimate $x_k$ depend on measurement up through the present value $y_{k-1}$. The fixed gain is:

$$L = PC'R^{-1}$$

where $P$ satisfies $AP + PA' - PC'R^{-1}CP + BQB' = 0$ (29)

Then we have:

$$\hat{x}(t) = A\hat{x}(t) - L(C\hat{x}(t) - y(t))$$

$$P_{k+1} = (F - L_k H)P_k^+ F' + GWG'$$

Initialization from (4) $x(0) = \hat{x}_0$ (31)

The mean square error $\epsilon(t) = E\{(x(t)-\hat{x}(t))'(x_k - \hat{x}(t))\}$ (32)

At time $t$, the error is determined by $\hat{x}_0$ and $L$ which is influenced by $Q$ and $R$. Choosing $L$ according to (27) and (28), $\epsilon_k$ is minimized as $k \to \infty$. Implement on discrete time stamp

$$\hat{x}_{k+1} = \hat{x}_k - T(A\hat{x}_k - L(C\hat{x}_k - y_k))$$ (33)
GENERATE CONTINUOUS AND DISCRETE MEASUREMENT SIGNALS

Continuous-time response of state variables are obtained using (1) with 1μs time step. Discrete-time measurement from (9) are obtained by using the function randn in Matlab to add sensor noise with sampling period T = 1ms.

\[ Q = 10^6, R = \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1.6 \end{bmatrix}, \quad W = 10^9, \quad V = \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1.6 \end{bmatrix}, \quad \hat{x}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad P_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

Fig. 5. Simulation of tag motion and sensor measurements. The red solid curve represents the actual motion and the blue dots represent the samples of sensor outputs.
$Y_{pos}$ is the vector of sensed values, $X_{pos}$ is the vector of true values, and $\hat{X}_{pos}$ is the vector of estimated values.

$$E_{est} = X_{pos} - \hat{X}_{pos}$$

RMS error is $E_{rms} = \left(\frac{1}{N} \sum_{n=1}^{N} E^2[n]\right)^{\frac{1}{2}}$ and peak error is: $E_{peak} = \max\{|E[1]|, \ldots, |E[N]|\}$
ESTIMATION ACCURACY

• The first table shows the poor sensor accuracy whereas the second table shows the excellent estimator accuracies.

• The estimators are 89 times more accurate than the sensors with respect to the peak errors, whereas they are 47 times more accurate with respect to rms errors.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>POSITION SENSOR PERFORMANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{out,peak}}$</td>
<td>800 mm</td>
</tr>
<tr>
<td>$E_{\text{out,rms}}$</td>
<td>223 mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>POSITION ESTIMATOR PERFORMANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design #1</td>
<td>Design #2</td>
</tr>
<tr>
<td>$E_{\text{est,peak}}$</td>
<td>9.05 mm</td>
</tr>
<tr>
<td>$E_{\text{est,rms}}$</td>
<td>4.74 mm</td>
</tr>
</tbody>
</table>
CONCLUSION

• The addition of a Kalman filter to the original HIMR algorithm reduces estimation errors of motion variables to around 5mm rms.

• The fixed-gain version of Kalman filter is more computationally efficient than the variable-gain version in usage, yet has similar performance.
Thank you!