ECPE 3025 Homework 1: Phasors Smith Charts

Solutions

1. To show why this is true, start with the basic convolution integral and substitute a generic sinusoidal input $x(t) = A\cos(2\pi ft + \phi)$:

$$\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)\,d\lambda \\
&= \int_{-\infty}^{\infty} x(t-\lambda)h(\lambda)\,d\lambda \\
&= \int_{-\infty}^{\infty} A\cos(2\pi f[t-\lambda]+\phi)h(\lambda)\,d\lambda \\
&= A\left[\cos(2\pi ft)\underbrace{\int_{-\infty}^{\infty}\cos(\phi-2\pi f\lambda)h(\lambda)\,d\lambda}_{H_x} - \sin(2\pi ft)\underbrace{\int_{-\infty}^{\infty}\sin(\phi-2\pi f\lambda)h(\lambda)\,d\lambda}_{H_y}\right] \\
&= A\left[H_x\cos(2\pi ft) - H_y\sin(2\pi ft)\right] \\
&= A\sqrt{H_x^2+H_y^2}\cos\left(2\pi ft - \tan^{-1}\frac{H_y}{H_x}\right)
\end{aligned}$$

After a series of manipulations, we see that the final answer is a sinusoid whose overall amplitude and phase depend on the system characteristics. Since H_x and H_y evaluate to constants, the result is *always* true for any LTI system; the frequency f does not change. You have just proven the most important applied theorem in electrical engineering.

2. Before we start, let's calculate the electromagnetic dimensions of the coaxial cable. The wavelength of radiation is

$$\lambda = \frac{v_p}{c} = \frac{1.8\times 10^8 \text{ m/s}}{5.75\times 10^9 \text{ Hz}} = 0.0313 \text{ m}$$

This makes the 4.25-cm coaxial cable about 1.3576λ in length.

(a) Reflection coefficient calculation:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(25-j75)-50}{(25-j75)+50} = 0.3333 - j0.6667 = 0.75\angle-63.4^\circ$$

(b) Using our basic transmission line formula:

$$Z_{eq} = Z_0\left[\frac{Z_L + jZ_0\tan\beta D}{Z_0 + jZ_L\tan\beta D}\right] = 55.8 + j*118.0\,\Omega$$

(c) The normalized load impedance is $0.5 - j1.5$ and must be rotated 1.3576λ about the center to find Z_{in}. The final destination is a normalized impedance of $1.12 + j * 2.36$ which results in $55.8 + j * 118.0\,\Omega$. See the Smith Chart sketch in Figure 1.

(d) Below is the full wave solution for the transmission line:

$$\begin{aligned}\tilde{v}(z) &= V^+ \exp(-j\beta z) + V^- \exp(j\beta z)\\ \tilde{i}(z) &= \frac{V^+}{Z_0}\exp(-j\beta z) - \frac{V^-}{Z_0}\exp(j\beta z)\end{aligned}$$

We are not given the voltage of excitation, but we are given most of the other values:

$$\begin{aligned}\tilde{v}(z) &= V^+ \exp(-j200.7z) + V^- \exp(j200.7z)\\ \tilde{i}(z) &= \frac{V^+}{50}\exp(-j200.7z) - \frac{V^-}{50}\exp(j200.7z)\end{aligned}$$

For the truly motivated student, recognize that V^- is related to V^+ *at the end of the line* by the reflection coefficient computed in (a). Thus, we could write:

$$\begin{aligned}\tilde{v}(z) &= V^+ \exp\left(-j77.0\left[z - .0425\right]\right) + 0.75V^+ \exp\left(j\left[77.0(z - .0425) - 63.4^\circ\right]\right)\\ \tilde{i}(z) &= \frac{V^+}{50}\exp\left(-j77.0\left[z - .0425\right]\right) - 0.0150V^+ \exp\left(j\left[77.0(z - .0425) - 63.4^\circ\right]\right)\end{aligned}$$

which goes above and beyond a full-credit answer.

3. Refer to Figure 2 for a simplified circuit diagram of the damaged line. Here the 75Ω load, since it is still matched to the last section of cable, is transformed without change to the point of damage. Now we need to calculate the equivalent load at this point – a 200Ω resistance in parallel with a capacitor and the 75Ω resistance – in order to transform the impedance further up the line:

$$Z_{eq} = Z_0\left[\frac{Z_L + jZ_0\tan\beta D}{Z_0 + jZ_L\tan\beta D}\right] \qquad Z_L = \frac{(120)(75 - j\frac{1}{2\pi fC})}{120 + 75 - j\frac{1}{2\pi fC}}$$

Once Z_{eq} is known, the voltage and current at the front of the line are given by the voltage-divider equation:

$$V_S = V_{source}\frac{Z_{eq}}{Z_0 + Z_{eq}} \qquad I_S = \frac{V_{source}}{Z_0 + Z_{eq}}$$

The general voltage-current relationships on the transmission line are given by the following system of equations:

$$\begin{aligned}\tilde{v}(z) &= V^+ \exp(-j\beta z) + V^- \exp(j\beta z)\\ \tilde{i}(z) &= \frac{V^+}{Z_0}\exp(-j\beta z) - \frac{V^-}{Z_0}\exp(j\beta z)\end{aligned}$$

We can always solve for V^+ and V^- by setting $\tilde{v}(0)$ and $\tilde{i}(0)$ equal to V_S and I_S respectively. This results in the following relationships:

$$V^+ = \frac{V_S + I_S Z_0}{2} \qquad V^- = \frac{V_S - I_S Z_0}{2}$$

which may be plugged back into the transmission line equation to solve for voltage and current *at the end of the line.* Whatever current results (V_L and I_L) then divides across the load resistors.

Since the problem requests the computation to be made at several frequencies, it is probably easiest to enter the equations into a computer script using a software like MatLab™ and re-run the code with different values of frequency. Assuming that the source voltage has amplitude 1V (RMS), here is the power that is absorbed by the television load for each case:

Case	Power (mW)	Loss (dB)
Undamaged	3.3	–
100 MHz	2.1	2.1 dB
400 MHz	2.3	1.5 dB
900 MHz	2.4	1.5 dB

Note that the loss is frequency dependent. This is interesting for the application of cable television signals, since analog television signals are frequency-division multiplexed across the spectrum between DC and 1 GHz. Thus, the damage will not affect all channels evenly.

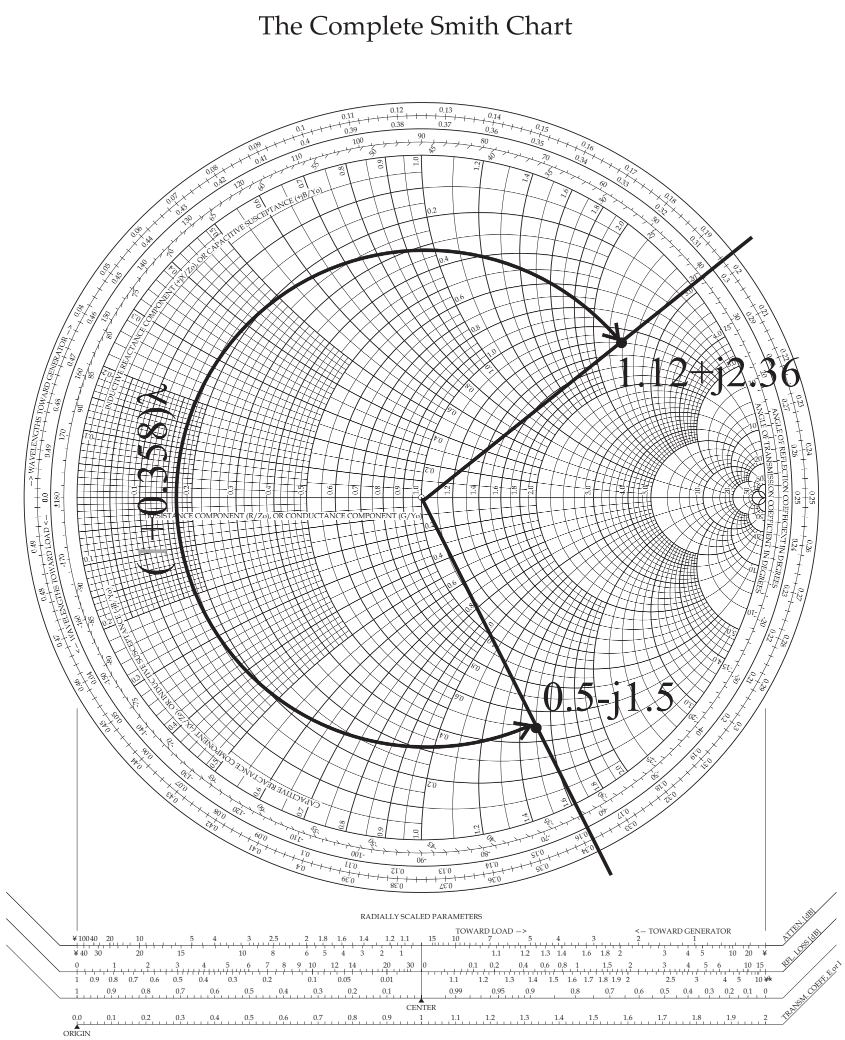

Figure 1: Smith Chart sketch for problem 2c.

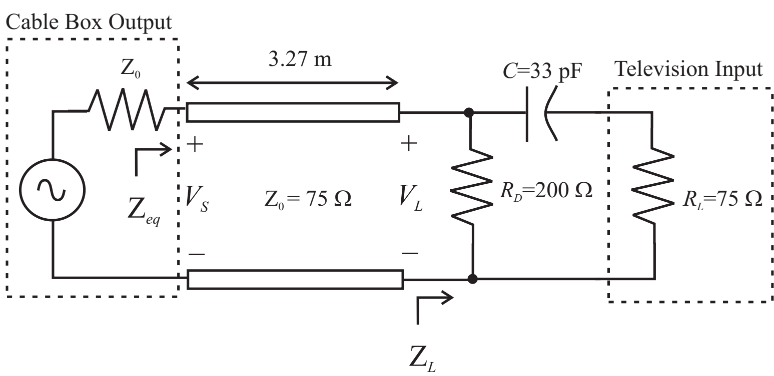

Figure 2: Equivalent circuit diagram for Problem 3.