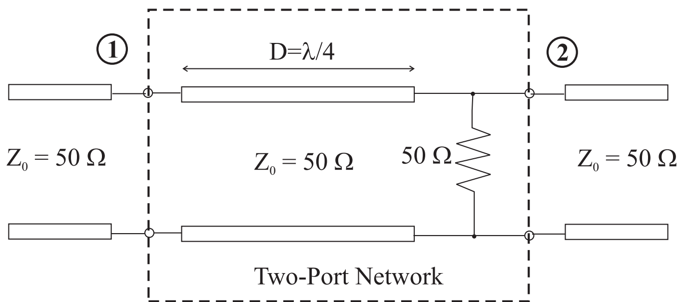
ECE 3065 Homework 8: S-parameters

Due Date: Never

1. **Calculate S-Parameters:** Calculate the S-matrix for the two-port circuit below (5 points):



2. **Three-Port to Two-Port Conversion:** You are given a three-port device with known S-matrix and convert it to a two-port device by placing a fixed load on the third port. This produces a new 2×2 S-matrix. For example, if you short the third port, the S-matrix for the new two-port circuit becomes:

$$S' = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} - \frac{1}{1 + s_{33}} \begin{bmatrix} s_{31}s_{13} & s_{31}s_{12} \\ s_{32}s_{13} & s_{23}s_{32} \end{bmatrix}$$

(See hint on next page.) Answer the following questions: (10 points)

(a) Write the new two-port S-parameter matrix if a matched 50Ω load is attached at port 3.

(b) Write the new two-port S-parameter matrix if port 3 is kept open-circuited.

(c) Write the new one-port S-parameter matrix (just s_{11}) if port 2 is connected to port 3 with an electromagnetically short 50Ω cable.

Hint for Problem 2: Short Circuit Solution
Start with the definition of a 3-port S-matrix:

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \end{bmatrix}$$

If port 3 is shorted, then the total voltage at the terminal must be zero, implying that $V_3^+ + V_3^- = 0$. From the S-matrix, we can write

$$\begin{aligned} V_3^- &= s_{31}V_1^+ + s_{32}V_2^+ + s_{33}V_3^+ \\ -V_3^+ &= s_{31}V_1^+ + s_{32}V_2^+ + s_{33}V_3^+ \end{aligned}$$

Solving for V_3^+, we find that

$$V_3^+ = -\frac{1}{1+s_{33}} \left[s_{31}V_1^+ + s_{32}V_2^+ \right]$$

Now let us rewrite the top two columns of the S-matrix:

$$\begin{aligned} \begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} &= \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \end{bmatrix} \\ &= \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix} + \begin{bmatrix} s_{13} \\ s_{23} \end{bmatrix} V_3^+ \\ &= \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix} - \frac{1}{1+s_{33}} \begin{bmatrix} s_{13} \\ s_{23} \end{bmatrix} \begin{bmatrix} s_{31} & s_{32} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix} \\ &= \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix} - \frac{1}{1+s_{33}} \begin{bmatrix} s_{13}s_{31} & s_{13}s_{32} \\ s_{23}s_{31} & s_{23}s_{32} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix} \\ &= \underbrace{\left(\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} - \frac{1}{1+s_{33}} \begin{bmatrix} s_{13}s_{31} & s_{13}s_{32} \\ s_{23}s_{31} & s_{23}s_{32} \end{bmatrix} \right)}_{S'} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix} \end{aligned}$$

Whew! All that work to go from 3 ports to 2 ports.